

Control Topologies for Deep Space Formation Flying Spacecraft

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Abstract

A formation of satellites flying in deep space can be specified in terms of the relative satellite positions and absolute satellite orientations. The redundancy in the relative position specification generates a family of control topologies with equivalent stability and reference tracking performance. This paper gives a characterization of the equivalent topologies and uses this approach to show that there exists a control topology which achieves a global tracking objective using only local controllers. This is referred to as a local relative topology and can be implemented without requiring communication between the spacecraft in the formation.

1 Introduction

The collective behavior of spacecraft flying in formation can be used to synthesize instruments of greater utility than could otherwise be achieved with a single spacecraft. One example—which motivates much of our work—is an interferometric imaging system composed of multiple spacecraft. Preliminary work on orbiting

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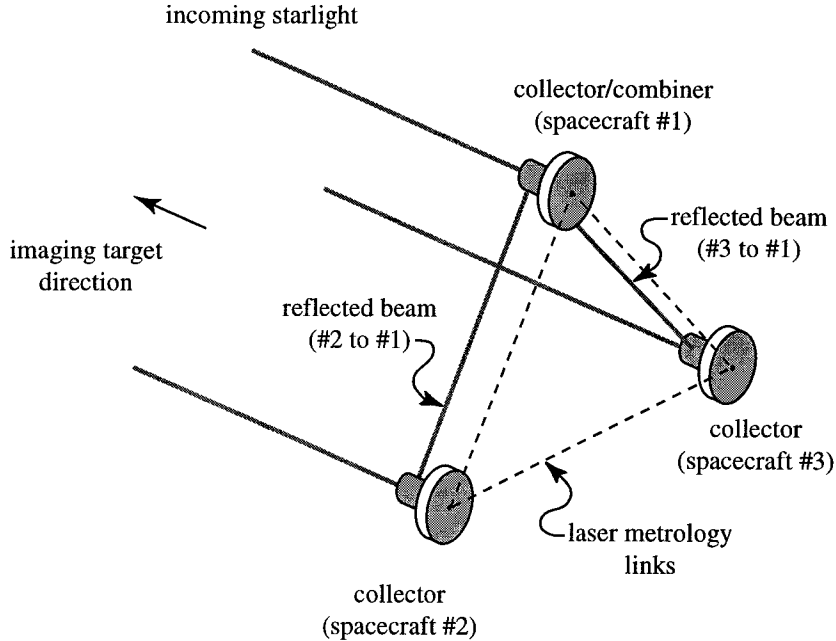


Figure 1: Interferometric imaging configuration using multiple spacecraft in formation. Spacecraft separations are of the order of tens to hundreds of meters.

interferometers can be found in [1, 2, 3]. Several interferometric flight projects, using formation flying, have been proposed and studied including LISA [4], Terrestrial Planet Finder (TPF) [5] and Starlight (formerly ST-3) [6].

We use the interferometric imaging application as a basis for discussing formation control problems which are applicable to a wider range of problems. Figure 1 illustrates a conceptual interferometric imaging configuration.

Each spacecraft acts as a collector, reflecting light from the imaging source to a combiner spacecraft. The light from any two collectors is combined at a detector and, if the optical pathlengths are held fixed, an interference pattern can be measured. Each measurement of the amplitude and phase of this pattern amounts to a sample of the spatial Fourier transform of the image. Multiple measurements, using either multiple collectors simultaneously or repositioning fewer collectors, gives sufficient data to allow reconstruction of the image. The advantage of imaging in this way is that the effective aperture depends on the collector separation. Future

objectives call for separations of the order of kilometers, giving resolutions that cannot be matched by any individual spaceborne telescope. Multiple collectors can also be used to create nulls in the spatial response of the array thereby enabling the imaging of dim objects adjacent to bright ones. This is a promising technology for searching for planetary objects in other solar systems. There are many contributions in the literature on interferometric imaging; see [7] for illustrative examples.

Formation control problems—initialization, reorientation, resizing, tracking, station keeping, etc.—can be specified in terms of the tracking of relative spacecraft position and spacecraft attitude. The complete control problem is likely to be hierarchical, with a supervisory controller specifying the required formation during maneuvers or science data collection.

In interferometric applications the control actuation will also be hierarchical. The tight constraint on the optical path length is achieved by controlling the spacecraft position to the order of tens of micrometers. Each spacecraft will have the optical components mounted on a movable platform, which can be positioned to the order of micrometers or less. The individual mirrors themselves are mounted on precision piezoelectric actuators which in turn can be controlled to the order of tens of nanometers. This is the requirement in order to generate interference patterns between the optical beams.

The range of operation, and the bandwidth, of each of these actuation systems varies widely. The actuator allocation design will depend on the specific details of the configuration. In this paper we consider only generic actuation, and assume that the control system is able to exert a force suitable for positioning in three dimensions. Our emphasis is on the control topology at the level of communication between the spacecraft, rather than the nested actuation hierarchy within a spacecraft.

Our work focuses on deep space missions, where the formation is in heliocentric orbit rather than earth orbit. The analysis results we present are quite general; however in applying them to our formation flying problem we make some specific assumptions. The most significant of these is that the spacecraft can sense their relative position and not their absolute position.

The spacecraft in the formation are free flying and their dynamics are coupled only through the application objectives and relative measurements of the spacecraft positions and velocities. To maintain the performance of the formation, in deep

space missions, it is necessary to maintain the relative position and absolute orientation of the spacecraft. Actuation for control purposes is performed on the individual spacecraft.

There are many possible topologies for the sensing, control, and communication within the formation. Communication bandwidths, synchronization constraints, and sensor capabilities affect the performance of any chosen topology. These issues have been studied; see, for example, [8, 9, 10, 11, 12, 13, 14, 15] for work on leader/follower and other topologies, and [16] on estimation configurations.

We can speak of centralized or decentralized topologies for both the control design and implementation. A centralized—or global—control design topology is one in which the actuation is calculated from information or measurements of all formation variables. A decentralized design implies that the control actuation for each spacecraft depends only on a subset of the formation variables. Decentralization is a matter of degree, and can be used to trade-off between formation performance and controller/communication complexity. See [17] and the references therein for a discussion of decentralized control in spacecraft formations.

Centralized or decentralized topologies may also be considered in the implementation. In a centralized implementation all measurements would be sent to a single spacecraft, the required actuation calculated, and then communicated back to the individual formation members. An equivalent controller can also be implemented by having each spacecraft maintain a copy of that part of the controller which generates its actuation. Measurement information is then transmitted through the formation for use by the individual controllers.

Communication bandwidth and synchronization constraints make it advantageous to reduce the communication of time critical information required between spacecraft. In this context we consider the following problem. Is there an implementation topology which allows a centralized/global control design topology to be implemented with a minimum of communication between the spacecraft? We develop tools which allow this problem to be addressed, and use these to show that there exists an implementation topology for global/centralized control designs that requires *no* communication between spacecraft. Only local measurements are required for the implementation, which makes the approach suitable for small to medium formations (3 to 6 spacecraft).

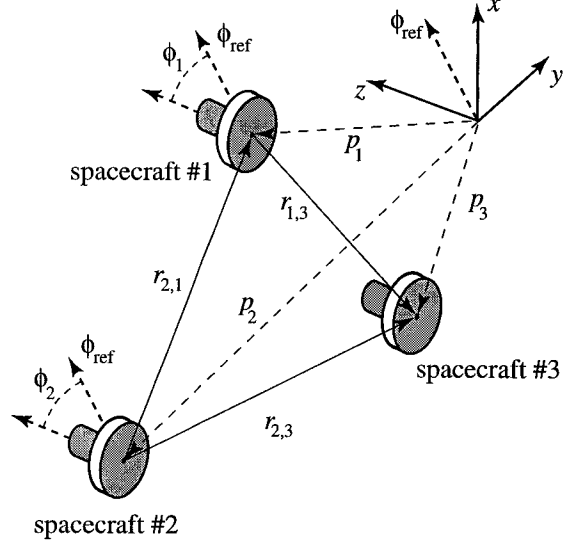


Figure 2: Spacecraft formation: the local and relative position variables are shown.

2 Formation definition and sensing

We begin by considering a typical formation and defining the notation associated with the various local and relative position and attitude variables. Consider a formation of N spacecraft, and for simplicity it is sufficient here to define on each a reference attitude, ϕ_i , $i = 1, \dots, N$, with respect to an inertially fixed direction, ϕ_{ref} . Figure 2 illustrates these definitions.

We define a local inertial frame within which each spacecraft is located at position $p_i = [x_i, y_i, z_i]'$ (the prime denotes transpose). The origin of this frame is not critical for the application we consider here. The relative position between each two spacecraft is defined as,

$$r_{ij} = p_j - p_i = \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \quad i, j = 1, \dots, N, \quad i \neq j.$$

Naturally, $r_{ij} = -r_{ji}$, and in an N spacecraft formation there are $N(N-1)/2$

relative three dimensional distances that can be defined modulo the opposite direction equivalences.

In deep space, an accurate measurement of (x_i, y_i, z_i) is not available. It may be possible to obtain range and direction information with respect to Earth, but this will be accurate only to the order of meters.

On the other hand, the $r_{i,j}$ variables can be precisely measured. The Autonomous Formation Flying Sensor (AFF) [18] uses a GPS based architecture with several transmitters and receivers mounted on each spacecraft, to give relative position measurements to ± 10 mm, relative velocity measurements to ± 0.1 mm/sec. Laser metrology can be used to give relative measurements accurate to ± 10 nm or better.

In contrast to the absolute position, spacecraft attitude can be measured to very high accuracy. On-board star trackers are typically used to provide attitude information for each spacecraft, and these have a typical accuracy of ± 10 milliarcseconds (mas). Higher quality star trackers can provide an accuracy of less than ± 1 mas.

The formation is defined by the relative spacecraft positions and the attitude of each,

$$\begin{aligned} r_{i,j} &: i, j = 1, \dots, N, \ i \neq j \\ \phi_i &: i = 1, \dots, N. \end{aligned}$$

Accurate measurements of the above formation variables are available; an accurate measurement of the absolute location of the formation is not.

Note that there is some redundancy in the above as the $N(N-1)/2$ relative positions are not independent. We will exploit this redundancy in looking for control topologies that do not require all relative positions to be measured.

3 Formation control

Consider the global reference tracking/disturbance rejection control design problem illustrated in Figure 3. The dynamics of the formation are modeled in P where

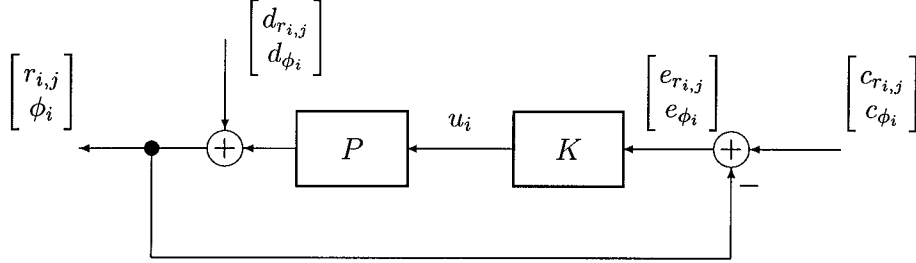


Figure 3: Control design formulation for formation reference tracking. Reference commands are denoted by $c_{r_{i,j}}, c_{\phi_i}$ and errors are denoted by $e_{r_{i,j}}, e_{\phi_i}$.

there inputs, u_i , the commanded thrusts for each spacecraft. The measurements are the formation variables, $r_{i,j}$ and ϕ_i . System disturbances are modeled here by $d_{r_{i,j}}, d_{\phi_i}$, adding directly to the formation variables. The analysis results given below apply equally to the case where the relative velocities, $\dot{r}_{i,j}$, are also measured, but these are neglected for notational simplicity.

Global control problems, such as that specified above, are readily handled by existing optimal control theory and supported by analysis and synthesis software. For example, K above may have been designed to meeting an \mathcal{H}_∞ or \mathcal{H}_2 /LQG objective for the formation.

3.1 Attitude control

The formation specification includes the attitude of each spacecraft, ϕ_i , with respect to a fixed inertial attitude. Because the formation dynamics are coupled only through the control objective, attitude errors, $\tilde{\phi}_i$, on each spacecraft are not coupled to attitude errors on the other spacecraft.

In the deep space mission application, each spacecraft is also assumed to have a local measurement of its attitude. This means that correcting attitude errors is a strictly local control problem: only local measurements are required to determine $\tilde{\phi}_i$, and only local actuation is required to attenuate the attitude error. Control of ϕ_i is

therefore decentralized, both in terms of design and implementation. For this reason we drop control of ϕ_i from further consideration and focus on the more difficult problem of the control of $r_{i,j}$.

3.2 Relative position control

The full set of relative position measurements contain redundancies that can be expressed as algebraic constraints. For example,

$$r_{i,j} + r_{j,k} + r_{k,i} = 0.$$

For the formation to be well defined these constraints must also apply to the relative position commands, $c_{r_{i,j}}$. Note that they therefore also apply to the errors and disturbances, $e_{r_{i,j}}$, $d_{r_{i,j}}$.

Express this constraint in the form,

$$\begin{bmatrix} r_{1,2} \\ r_{1,3} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = C \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} -I & I & 0 & \cdots & 0 \\ -I & 0 & I & \cdots & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & \cdots & & -I & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}.$$

The matrix $C \in \mathcal{R}^{3N(N-1)/2 \times 3N}$, and in a state-space representation may actually be a submatrix of the state to output matrix. It has rank $3(N-1)$ which means that it has a $3(N-1)(N-2)/2$ dimension null space. There exists a matrix, $M \in \mathcal{R}^{3N(N-1)/2 \times 3(N-1)(N-2)/2}$ satisfying,

$$M' \begin{bmatrix} r_{1,2} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = 0,$$

or equivalently, $M'Pu = 0$ for all u . This is a convenient method of expressing the algebraic redundancies in the relative position measurements.

4 Equivalent topologies

We will use the redundancy characterized above to define a class of transformation matrices which will have the effect of removing specified relative measurements from the controller. This class of transformation matrices is defined by,

$$H = I - XM',$$

where $X \in \mathcal{R}^{3N(N-1)/2 \times 3(N-1)(N-2)/2}$ and satisfies,

$$M'X = I.$$

Tedious algebra shows that this has the effect of expressing some of the relative position measurements as linear combinations of the others.

To clarify this we give a three spacecraft example. In this case,

$$\begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} = \begin{bmatrix} -I & I & 0 \\ -I & 0 & I \\ 0 & -I & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}.$$

Amongst three spacecraft there are only two independent relative positions and this can be expressed as,

$$M'_1 \begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} = 0,$$

where $M_1 = \begin{bmatrix} I & -I & I \end{bmatrix}'$ is one choice. Now select $X_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}'$ and note that $M'_1 X_1 = I$. The transformation matrix, H_1 , is given by,

$$H_1 = I - X_1 M'_1 = \begin{bmatrix} 0 & I & -I \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

and this gives,

$$\begin{bmatrix} r_{1,3} - r_{2,3} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} = H_1 \begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix}.$$

Note that $r_{1,2}$ has explicitly been removed. This effect can be seen more clearly when we consider KH_1 for some controller K designed to use all three relative measurements,

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix}.$$

Define $\hat{K} = KH_1$ and consider the control action generated,

$$\begin{aligned} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} &= \hat{K} \begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} \\ &= \begin{bmatrix} 0 & K_{11} + K_{12} & -K_{11} + K_{13} \\ 0 & K_{21} + K_{22} & -K_{21} + K_{23} \\ 0 & K_{31} + K_{32} & -K_{31} + K_{33} \end{bmatrix} \begin{bmatrix} r_{1,2} \\ r_{1,3} \\ r_{2,3} \end{bmatrix} \\ &= \begin{bmatrix} K_{11} + K_{12} & -K_{11} + K_{13} \\ K_{21} + K_{22} & -K_{21} + K_{23} \\ K_{31} + K_{32} & -K_{31} + K_{33} \end{bmatrix} \begin{bmatrix} r_{1,3} \\ r_{2,3} \end{bmatrix}. \end{aligned}$$

The transformation, H_1 , has the effect of removing the measurement $r_{1,2}$ from the controller. There are other obvious choices for X_1 which would remove one of the other relative position measurements. The method is independent of the null space characterization, M_i . A different M_i would simply require a different X_i to achieve the same result.

It is not true that $KH_i = K$. However the next section will show that KH has the same stability and tracking performance properties as the global controller K .

5 Stability and performance analysis

Before proceeding we will extend our class of transformations slightly. Partition the identity into q block diagonal pieces via,

$$I = \sum_{i=1}^q E_i.$$

Now define the transformed controller via,

$$\hat{K} = \sum_{i=1}^q E_i K H_i,$$

where the H_i are transformations of the form, $H_i = I - X_i M_i'$. This has the effect of grouping the controller outputs into q disjoint groups, and applying a different input transformation, H_i , to each.

Lemma 1 *Given a full rank matrices, M_i satisfying $M_i' P = 0$, and matrices X_i , $i = 1, \dots, q$, satisfying $M_i' X_i = I$, define, $H_i = I - X_i M_i'$. Define q matrices E_i such that,*

$$I = \sum_{i=1}^q E_i.$$

Then,

$$\hat{K} = \sum_{i=1}^q E_i K H_i$$

satisfies

i) $\hat{K} y = K y$ for all y satisfying $M_i' y = 0$, $i = 1, \dots, q$.

$$ii) \hat{K}P = KP.$$

Proof: To begin note that $H_i y = (I - X_i M_i') y = y$. Part *i*) is shown as follows:

$$\hat{K}y = \left(\sum_{i=1}^q E_i K H_i \right) y = \left(\sum_{i=1}^q E_i \right) Ky = Ky.$$

Part *ii*) now follows from observing that for all y generated by $y = Pu$, satisfies $M_i' y = 0$, $i = 1, \dots, q$. ■

It is interesting to note that the E_i can be more general than disjoint partitions of the identity.

Because $\hat{K}P = KP$ the signals that go around the loop are the same for both controllers. This is the basis of the stability equivalence proven below.

Theorem 2 (Stability) *Given a full rank matrices, M_i satisfying $M_i' P = 0$, and matrices X_i , $i = 1, \dots, q$, satisfying $M_i' X_i = I$, define, $H_i = I - X_i M_i'$. Define q matrices E_i such that,*

$$I = \sum_{i=1}^q E_i.$$

Given a stable linear controller, K , that internally stabilizes the linear plant P , then

$$\hat{K} = \sum_{i=1}^q E_i K H_i,$$

also internally stabilizes P .

Proof: The internal stability of the (P, K) loop is equivalent to the input/output stability of the mappings:

$$\begin{array}{ll} i) & (I + PK)^{-1}P \\ iii) & (I + KP)^{-1}K \end{array} \quad \begin{array}{ll} ii) & (I + PK)^{-1}PK \\ iv) & (I + KP)^{-1} \end{array}$$

We now show the stability each of these relationships with \hat{K} in place of K . Part *i)* is the plant input disturbance to plant output mapping. For all plant input disturbances, x , the plant output satisfies $M'_i y = 0$ (by assumption), and

$$y = (I + P\hat{K})^{-1}Px \quad \text{or, equivalently,} \quad Px = (I + P\hat{K})y.$$

From Lemma 1, $\hat{K}y = Ky$ and so y also satisfies $Px = (I + PK)y$, which is a stable mapping by assumption. To prove Part *ii)* note that,

$$y = (I + P\hat{K})^{-1}P\hat{K}x = x - (I + P\hat{K})^{-1}x.$$

It is sufficient to show that the second term—the output disturbance response of the plant—is stable. This must satisfy, $M'_i y = 0$ and

$$x = (I + P\hat{K})y = (I + PK)y, \quad \text{by Lemma 1, part } i),$$

which is again stable by assumption.

We now consider part *iv)* which follows by noting that $(I + \hat{K}P)^{-1} = (I + KP)^{-1}$ (via Lemma 1, part *ii)*).

Part *iii)* is the mapping with signals x and u satisfying,

$$(I + \hat{K}P)u = \hat{K}x \quad \text{or, equivalently,} \quad (I + KP)u = \hat{K}x.$$

Note \hat{K} is an open-loop linear combination of stable transformations, H_i , and partitions of K . The stability of K therefore implies the stability of \hat{K} . Therefore $\hat{x} := \hat{K}x$ is bounded, and part *iv)* result implies the stability of part *iii)*. ■

The restriction to stable controllers can be relaxed; as stated it addresses the most practically applicable case.

Although the signals going around the loop in each case are the same, the performance transfer functions may differ as $P\hat{K} \neq PK$ for all possible inputs to K . In the formation flying case, we may assume that the commanded relative positions specify a feasible formation, which allows us to prove equivalence in reference tracking performance between the two controllers.

Theorem 3 (Performance) *Given a full rank matrices, M_i satisfying $M_i'P = 0$, and matrices X_i , $i = 1, \dots, q$, satisfying $M_i'X_i = I$, define, $H_i = I - X_iM_i'$. Define q matrices E_i such that,*

$$I = \sum_{i=1}^q E_i.$$

Given a stable, internally stabilizing controller, K , define

$$\hat{K} = \sum_{i=1}^q E_i K H_i.$$

For all reference inputs satisfying $M_i'c_r = 0$,

$$y = (I + PK)^{-1}PK c_r = (I + P\hat{K})^{-1}P\hat{K}c_r.$$

Proof: Note that $M_i'c_r = 0$ implies that for all H_i , $H_i c_r = c_r$. Consider the plant output, y , with \hat{K} in the loop.

$$y = (I + P\hat{K})^{-1}P\hat{K}c_r,$$

which implies that y satisfies,

$$(I + P\hat{K})y = P\hat{K}c_r.$$

By Lemma 1, part *i*) y and c_r also satisfy, $(I + PK)y = PKc_r$. ■

We have parametrized a set of controllers, \hat{K} , which have equivalent stability and tracking performance to a given global controller, K , using a characterization of the output null space. We note that we can develop a completely analogous theory based on an input null space characterization of the plant. In the formation flying application the input null space is of dimension three and this choice does not allow as much flexibility in zeroing out controller elements.

6 Local relative control topology

The analysis tools of the previous section can be used to illustrate the existence of a particularly interesting topology for formation flying. We define this as follows.

Definition 4 *A control topology in which all actuation signals depend only on relative measurements with respect to the actuation location is termed a local relative control topology.*

In our application this topology means that all control calculations can be performed locally, based only on local relative measurements. In other words, the calculation of the actuation, u_i , depends only on $r_{i,j}$, $j = 1, \dots, N$, $j \neq i$. It does not depend on $r_{k,j}$ when $k \neq i$ and $j \neq i$. This topology can be implemented without any communication between the spacecraft. We now give the main result.

Theorem 5 *Given a global control topology with a stable controller, K , with inputs being all relative position measurements, there exists a stabilizing local relative control topology, with stable controller \hat{K} , which has equivalent relative position tracking performance.*

Proof: We demonstrate this by constructing the local relative controller for spacecraft #1. The first $3(N - 1)$ elements of the measurement vector are the spacecraft #1 relative measurements, $r_{1,2}, \dots, r_{1,N}$. There are $3(N - 2)(N - 1)$ measurements which are not relative to spacecraft #1 and each of these satisfies,

$$r_{1,k} + r_{1,j} + r_{j,k} = 0, \quad j, k \neq 1.$$

Express these equations in matrix form,

$$\begin{bmatrix} Q & I \end{bmatrix} \begin{bmatrix} r_{1,2} \\ \vdots \\ r_{1,N} \\ r_{2,3} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = 0,$$

where $Q \in \mathcal{R}^{3(N-2)(N-1) \times 3(N-1)}$. Define $M_1 = \begin{bmatrix} Q & I \end{bmatrix}$ and note that the identity guarantees that M_1 is full rank. Now define,

$$X_1 = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad \text{and note that } M_1' X_1 = \begin{bmatrix} Q & I \end{bmatrix} \begin{bmatrix} 0 \\ I \end{bmatrix} = I.$$

Define $H_1 = I - X_1 M_1'$, noting that M_1 and X_1 satisfy the conditions of Theorems 2 and 3. Consider the result of,

$$H_1 r = \left(I - \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} Q & I \end{bmatrix} \right) \begin{bmatrix} r_{1,2} \\ \vdots \\ r_{1,N} \\ r_{2,3} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = \begin{bmatrix} I & 0 \\ -Q & 0 \end{bmatrix} \begin{bmatrix} r_{1,2} \\ \vdots \\ r_{1,N} \\ r_{2,3} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = \begin{bmatrix} I \\ -Q \end{bmatrix} \begin{bmatrix} r_{1,2} \\ \vdots \\ r_{1,N} \end{bmatrix}.$$

Observe that H_1 has removed all measurements that are not relative to spacecraft #1. Define the partition of the identity selecting the spacecraft #1 control outputs,

$$E_1 = \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & 0 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & 0 \end{bmatrix}.$$

Then E_1KH_1 gives the local relative controller for spacecraft #1. The other spacecraft relative controllers can be calculated similarly after reordering the relative measurement vector. The theorem now follows directly from Theorems 2 and 3. ■

By observing that the relative velocity measurements have the same null space as the relative position measurements we can extend Theorem 5 as follows.

Corollary 6 *Given a global control topology with a stable controller, K , with inputs being all relative position and velocity measurements, there exists a stabilizing local relative control topology, with stable controller \hat{K} , which has equivalent relative position and velocity tracking performance.*

The local relative topology has implementation advantages. The most significant is that it does not require measurement or state information to be communicated between the spacecraft. This can remove the need to synchronize the spacecraft timing at the control implementation level, and can remove one of the potential bandwidth constraints in the formation control problem. Note that some communication will still be required for supervisory tasks. In some hardware implementations the distinction between relative measurements and communications is not as unambiguous. For example the AFF [18] measuring system is based on communication signals. Some communication may also be required to initialize a laser ranging system between spacecraft.

It should be noted that this work addresses control topologies, and not estimation topologies. While the tracking response of each topology is equivalent, the noise response may not be. An estimator may derive additional benefit from non-local measurements.

The local relative topology is best suited for implementation with a small number of spacecraft. At some point the cost and complexity of a large number of relative measurements outweighs the disadvantages of communicating measurements and state variables between spacecraft.

The tools developed in proving the existence of the local relative topology are also applicable to other topological studies, as the next section illustrates.

7 Reconfiguration and other topologies

The characterization of the null space based transformations given here allows us to study other topologies, and topological reconfigurations.

Several situations may arise where control topology must be reconfigured during operation. Examples of this include failure of a sensing system and failure of a communication link. In such cases one or more of the measurements, $r_{i,j}$, is unavailable for control. A transformation matrix, H_j , can be calculated and applied to the global controller to recalculate a new equivalent topology that does not use the unavailable measurement. The new topology will, in general, require some communication of between the spacecraft. While this is simple to calculate “by hand”, the algebraic approach taken here allows for automatic recalculation in the case where multiple measurements are unavailable. Notice also that controller redesign is not required for this eventuality.

One important aspect of this is that the reconfiguration is algebraic rather than dynamic and therefore does not have any transient dynamics associated with it. For example, given two topologies, calculated by,

$$u_1(t) = KH_1y(t) \quad \text{and} \quad u_2(t) = KH_2y(t),$$

then the controller outputs $u_1(t) = u_2(t)$ for all t . This means that we can switch between controllers KH_1 and KH_2 without a transient in $u(t)$. Note that if the noise characteristics differ between the relative measurements then reconfiguration may change the formation noise response. It does not change the reference tracking characteristics.

Reconfiguration may be required as a result of the formation pattern itself. Consider the formation illustrated in Figure 4, where several spacecraft are aligned. If both the communication and relative sensing require line-of-sight contact the local relative topology cannot be implemented when the spacecraft move into this formation.

In this configuration only spacecraft #1 and #3 can implement local relative control. Under our assumptions $r_{2,4}$ cannot be measured from spacecraft #2. However, we can use the above analysis to show that it is sufficient for either

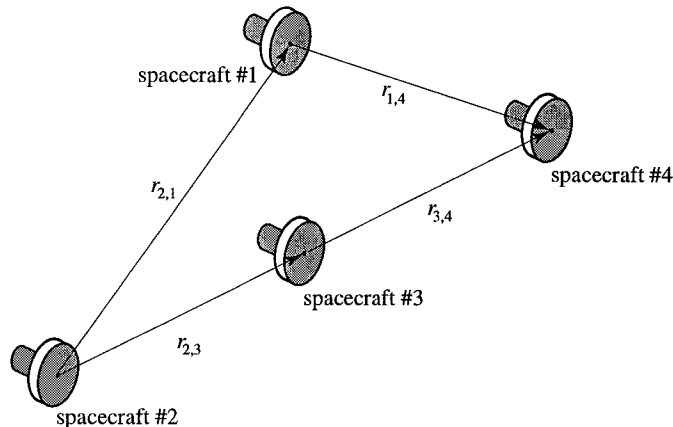


Figure 4: An example formation with spacecraft shadowing preventing full relative measurements.

spacecraft #1 to communicate $r_{1,4}$ or spacecraft #3 to communicate $r_{3,4}$ to spacecraft #2. An analogous situation exists for spacecraft #4.

We can use this approach to study nested topologies containing a mixture of relative measurements and communication. An example of this is illustrated in Figure 5. The highlighted spacecraft can execute its portion of the global control algorithm, without loss of formation tracking performance, if the relative measurements shown are communicated to it.

Clearly, there are many specific topological problems that can be considered in such formations and the work presented here provides tools for calculating equivalent topologies.

8 Conclusions

The techniques developed here exploit the redundancy in the plant measurement space to characterize controllers of different topologies with equivalent tracking and stability properties. The results are general, and are applied to study relative measurement configurations in formation flying spacecraft. In this case we have shown that there exists a local relative control topology which can be implemented

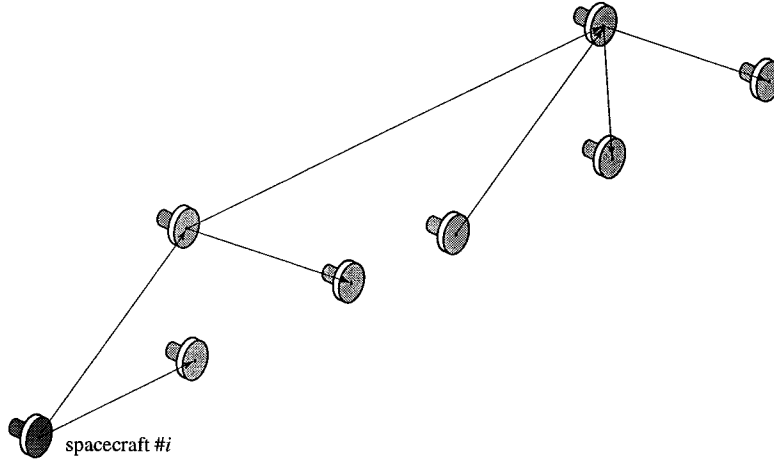


Figure 5: A nested topology. Spacecraft $\#i$ (highlighted) can execute an equivalent global algorithm based on the relative measurements shown.

using only local measurements on each spacecraft. The equivalence to a global controller means that more sophisticated global control design methods can be used, and then transformed into various equivalent topologies.

The controllers are equivalent from a reference tracking perspective. Their noise responses are not identical, and if the measurements are constructed by estimation techniques then the estimation topology may be different from the control topology.

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